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presented:

On the multicolor Ramsey number for 3-paths of length three

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Abstract

We show that if we color the hyperedges of the complete 3-uniform hypergraph on 2n + $\sqrt{18n+1}$ + 2 vertices with n colors, then one of the color classes contains a loose path of length three.

Definitions

- $Hypergraph$ H is a pair $H = (X, E)$ where X is a set of elements called nodes or vertices, and E is a set of non-empty subsets of X called hyperedges.
- *k-uniform hypergraph* is a hypergraph such that all its hyperedges have size k.
- *Hypergraph coloring* is assigning one of the colors from set $\{1, 2, 3, ...\lambda\}$ to every vertex of a hypergraph in such a way that each hyperedge contains at least two vertices of distinct colors. In other words, there must be no monochromatic hyperedge with cardinality at least 2.
- A proper edge-coloring of a hypergraph H with k colors is a function $c : E(H) \to \{1, \dots, k\}$ such that no two edges that share a vertex get the same color (number).
- P denotes the 3-uniform path of length three by which we mean the only connected 3-uniform hypergraph on seven vertices with the degree sequence $(2, 2, 1, 1, 1, 1, 1)$.
- C denotes the (loose) 3-uniform 3-cycle, i.e. the only 3-uniform linear hypergraph with six vertices and three hyperedges.
- F denotes the 3-uniform hypergraph on vertices $v1, v2, v3, v4, v5$ such that the first four of these vertices span a clique, and $v5$ is contained in the following three hyperedges: $v1v2v5$, $v2v3v5$, and $v3v4v5$.
- By $R(P; n)$ we denote the multicolored *Ramsey number* for P defined as the smallest number N such that each coloring of the hyperedges of the complete 3-uniform hypergraph K^3_n with n colors leads to a monochromatic copy of P.

Theorems

Theorem 1. $R(P; n) \leq 2n + \sqrt{18n + 1} + 2$

Lemma 1. Let H be a 3-uniform P-free hypergraph on $n \geq 5$ vertices. Then we can delete from H fewer than 3n hyperedges in such a way that the resulting hypergraph contains no copies of C and F.