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presented:

Approximately counting paths and cycles in a graph

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http://bit.ly/2zDDSVV

Problems

#PATHS is problem of counting paths in graph.
#CYCLES is problem of counting cycles in graph.

Definitions

- A counting problem $f : \Sigma^* \to \mathbb{N}$ is in #P if there is a non-deterministic polynomial-time Turing machine N such that for any string $x \in \Sigma^*$, the number of accepting paths of $N(x)$ is $f(x)$.
- A counting problem is #P-HARD if every counting problem in #P is reduced to the problem by a polynomial-time Turing reduction.
- A counting problem is #P-COMPLETE if the problem is in #P and is #P-HARD.
- A *randomized approximation scheme* (*RAS* for short) for a counting problem is a randomized algorithm which, given a string $x \in \Sigma^*$ and a real number $\epsilon > 0$, outputs $A(x)$ such that $Pr\{|A(x) - OPT(x)| \leq \epsilon \cdot OPT(x)\} \geq 3/4$.
- A *fully polynomial-time scheme* (*FPRAS* for short) for a counting problem is a *RAS* which runs in time polynomial in the size of x and ϵ^{-1}.
- Let $f, g : \Sigma^* \to \mathbb{N}$ be an arbitrary counting problems. An *approximation-preserving reduction* (*AP reduction* for short) from f to g is probabilistic oracle Turing machine M that takes as input a pair $(x, \epsilon) \in \Sigma \times (0, 1)$, and satisfies the following three conditions:
  1. Every oracle call made by M is of the form (ω, δ), where $\omega \in \Sigma^*$ is na instance of g, and δ is an error bound satisfying $\delta^{-1} \leq poly(|x|, \epsilon^{-1})$.
  2. The Turing machine M meets the specification for beeing *RAS* for f whenever the oracle meets the specification for being *RAS* for g.
  3. The running time of M is polynomial in $|x|$ and ϵ^{-1}.

  If there is an approximation-preserving reduction from f to g, then we write $f \leq_{AP} g$. If $f \leq_{AP} g$ and $g \leq_{AP} f$, then we write $f \equiv_{AP} g$.

Facts and Corollaries

Fact 1. #HAMPATHS is #P-COMPLETE, moreover #STHAMPATHS is #P-COMPLETE.
Fact 2. #HAMCYCLES is #P-COMPLETE.
Corollary 1. #HAMPATHS $\equiv_{AP}$ #SAT, moreover #STHAMPATHS $\equiv_{AP}$ #SAT.
Corollary 2. #HAMCYCLES $\equiv_{AP}$ #SAT.

Theorems

Theorem 1. #PATHS is #P-HARD.
Theorem 2. #CYCLES is #P-HARD.
Theorem 3. #PATHS $\equiv_{AP}$ #SAT.
Theorem 4. #CYCLES $\equiv_{AP}$ #SAT.