Session #6

Fundamentals of Microeconomics

INTRODUCTION TO GAME THEORY

November 11, 2019

Game theory allows us to model interaction of market participants

- Some motivation:

  http://www.youtube.com/watch?v=yM38mRHY150

- Game theory
- Static games
- Dynamic games

Game theory describes strategic behavior of players maximizing their payoff

- Game = competition between players, where strategic behavior has major importance.
- Action = move made by a player in a given stage of the game.
- Strategy = list of actions that a player will perform based on information in a given stage of a game, prepared for all possible situations.
- Payoff = monetary evaluation of a game.
- Strategic behavior = set of actions that the player performs to maximize his/her payoff, taking into account actions of the other plyers.

In game theory, we try to describe the game and then predict its outcome

- In a game, optimal strategy of each player depends on the behavior of others - we talk about mutual strategic dependence.
- Description of a game consist of description of players, rules, possible outcomes, payoffs related to these outcomes, and information that players have.
- Rules of a game include timing of individual moves and list of actions that players can perform in each move.
- We recognize static games (players make their moves together and only once) and dynamic games (players move consequently or repeatedly in several periods).

- Game theory
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STATIC GAMES CONSIST OF SIMULTANEOUS AND NON-REPEATED MOVES

- In static games, all players move simultaneously and only once.
- Players decide their strategies at the same time.
- Each player choses the strategy that maximizes his/her payoff.
- We assume that each player knows how his/her opponents should behave.

We usually describe static games in normal form, using the table of payoffs

- One of possible descriptions of a static game is the *normal form*: payoffs for different outcomes are written in a table where rows correspond to strategies of the first player and columns to strategies of the second player.
- Every combination of actions of the two players leads to a possible outcome corresponding to a pair of payoffs (x, y), where x is the payoff of the first player and y the payoff of the second player.
- Example: two companies (United Airlines and American Airlines) are competing for passengers on the line Los Angeles - Chicago. Here we understand by actions the amount of tickets sold and by payoffs the profits of the two firms.

We usually describe static games in normal form, using the table of payoffs

- We suppose that each of the companies has only 2 options - transport 64 or 48 thousands of passengers per quarter.
- Profits of the two companies (in millions USD) can be written in a table for all possible combinations.

		American Airlines	
		$q_A = 64$	$q_A = 48$
United Airlines	$q_U = 64$	4.1 , 4.1	5.1 , 3.8
	$q_U = 48$	3.8 , 5.1	4.6 , 4.6

To solve the game, we use the concept of Nash equilibrium

- To solve a game means to predict the result to which the game leads.
- This result has to be an equilibrium where none of the players is motivated to change his/her action if others do not change theirs.
- Solution can be found by the method of optimal response.
- Optimal response is the one that maximizes the payoff of the player given his/her beliefs about the opponents' strategy.
- Such solution of a game is called *Nash equilibrium*.
- We will illustrate this concept on the example of competing airlines.

We find the Nash equilibrium using the method of optimal response

- The first company (UA) optimizes its response to both possible actions of the second company:

		American Airlines	
		$q_A = 64$	$q_A = 48$
United Airlines	$q_U = 64$	<u>4.1</u> , 4.1	<u>5.1</u> , 3.8
	$q_U = 48$	3.8 , 5.1	4.6 , 4.6

- The second company (AA) optimizes its response to both possible actions of the first company:

		American Airlines	
		$q_A = 64$	$q_A = 48$
United Airlines	$q_U = 64$	4.1 , <u>4.1</u>	5.1 , 3.8
	$q_U = 48$	3.8 , <u>5.1</u>	4.6 , 4.6

We find the Nash equilibrium using the method of optimal response

- The result of the intersection of the optimal responses is the Nash equilibrium ($q_U = q_A = 64$):

		American Airlines	
		$q_A = 64$	$q_A = 48$
United Airlines	$q_U = 64$	4.1 , 4.1	5.1 , 3.8
	$q_U = 48$	3.8 , 5.1	4.6 , 4.6

- In this equilibrium, none of the companies wants to deviate (change its action), because, given the action of its opponent, it would get lower profit.
- Note that this equilibrium is not cooperative - it does not maximize the joint payoff.

COOPERATIVE EQUILIBRIA CANNOT BE ATTAINED IN ALL GAMES

- The equilibrium from the previous game is not cooperative - it has a structure of the so called *Prisoner's dilemma* game (see Exercise 1).
- In static games, cooperative equilibria can be attained if the structure of payoffs is favorable - such that it does not motivate players to deviate from the equilibrium (see Exercise 2).
- In other cases, cooperation is not possible because of mistrust - players do not believe that their opponents will not deviate.
- Cooperation can by enforced, but only in dynamic games.

EXERCISE 1 - Prisoner's dilemma

Suppose that two prisoners are arrested and they are interrogated separately. They can either confess (claim that they are both guilty) or deny (claim that they are innocent). What will be the outcome of this game if the table of payoffs is the following? (The numbers represent number of years in prison, negative signs signal the disutility of the two criminals.)

		Prisonner 2: Denies	Prisonner 2: Confesses
Prisonner 1	Denies	-1 , -1	-3 , 0
	Confesses	0 , -3	-2 , -2

Prisonner's dilemma explained by John Nash:
`http://www.youtube.com/watch?v=FdAIXil-ttE`

EXCERCISE 2

Suppose that two companies compete and both can decide whether to invest in advertising or not. Find the Nash equilibrium of this game for two possible table of payoffs:

1.

		Company 2	
		No advertising	Advertising
Company 1	No advertising	2 , 2	0 , 3
	Advertising	3 , 0	1 , 1

2.

		Company 2	
		No advertising	Advertising
Company 1	No advertising	2 , 2	3 , 4
	Advertising	4 , 3	5 , 5

EXCERCISE 3

Suppose that two companies (United Airlines and American Airlines) are competing for passengers on the line Los Angeles - Chicago. Each of the companies has only 3 options - transport 96, 64 or 48 thousands of passengers per quarter. Profits of the two companies (in millions USD) can be written in a table for all possible combinations. What will be the outcome of this game?

		American Airlines		
		$q_A = 96$	$q_A = 64$	$q_A = 48$
	$q_U = 96$	0 , 0	3.1 , 2.0	4.6 , 2.3
United Airlines	$q_U = 64$	2.0 , 3.1	4.1 , 4.1	5.1 , 3.8
	$q_U = 48$	2.3 , 4.6	3.8 , 5.1	4.6 , 4.6

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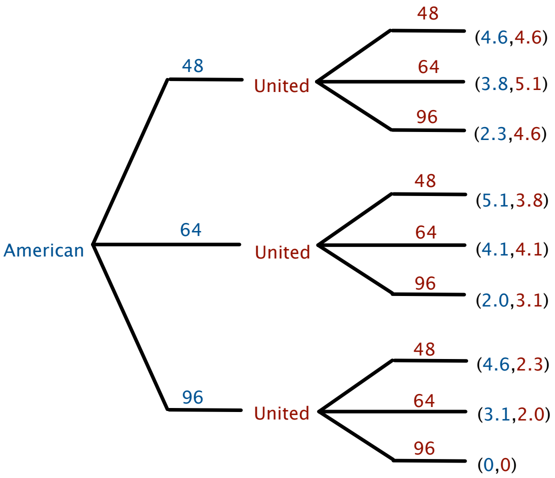
DYNAMIC GAMES CONSIST OF SEQUENTIAL OR REPEATED MOVES

- In dynamic games, players move sequentially (one after the other), or simultaneously but repeatedly.
- At each moment of the game, the player that is about to move has a perfect information about previous moves by all other players.
- We usually describe dynamic games in *extensive form*, where all players are specified, as well as all actions that they can perform in each move and pay-offs for all possible strategies.

We usually describe sequential games using a tree representing the sequence of decisions

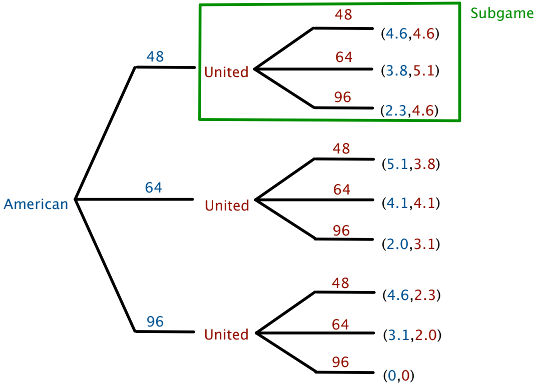
- We can revise the example of two airline companies from Application 3. Now we will suppose that one of the companies (American airlines) will decide first about its production and the second one (United airlines) will follow.
- We represent this game using a tree where each of the vertices represent a decision of one of the firms and edges represent a complete list of actions that firms can take.
- Payoffs are given in final vertices of the tree.

We usually describe sequential games using a tree representing the sequence of decisions



WE SOLVE SEQUENTIAL GAMES BY SEARCHING FOR EQUILIBRIA IN SUBGAMES

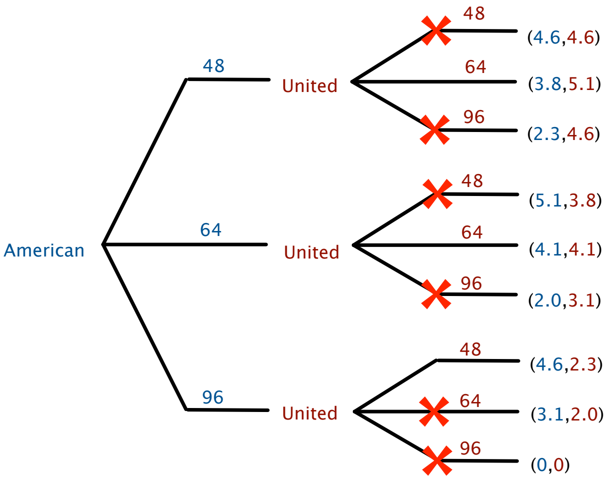
- A subgame consist of all possible actions at a given moment of the game and it is conditioned by previous decision of players:



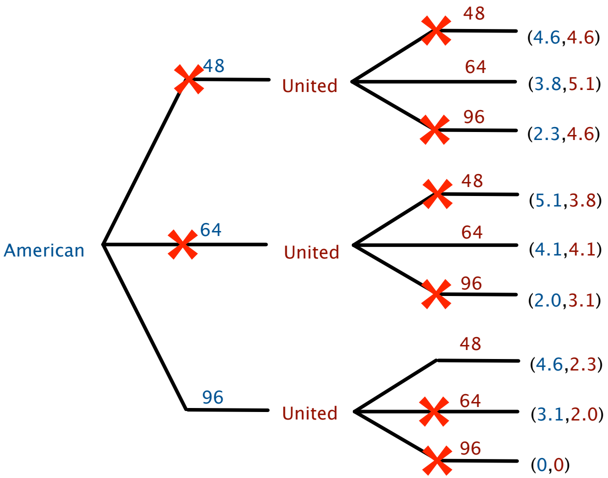
We solve sequential games by searching for equilibria in subgames

- To find the solutions, we need to find a *subgame perfect Nash equilibrium*, which is the situation when players' strategies represent Nash equilibrium in each subgame.
- To find such equilibrium, we use backward induction: first we find optimal strategy of the last player, then the strategy of the before last player, and so on, up to the first player.
- This means that at each vertex of a given level we eliminate all actions that do not lead to maximum payoff, given what will happen in the subsequent moves.
- We proceed so from the last to the first vertex of the game.

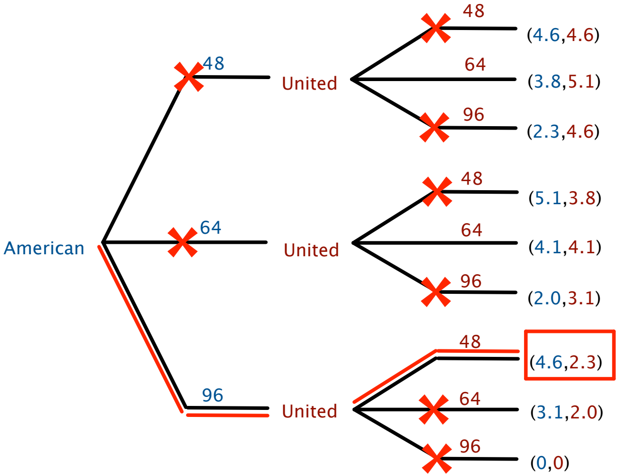
First we eliminate all actions not leading to maximum payoff in vertices in final level



Then we eliminate all such actions in previous levels of the game



Actions that remain after elimination represent subgame perfect Nash equilibrium



In sequential games, credible threats can be realized

- In order to achieve subgame perfect Nash equilibrium, each player has to believe that his/her opponents will chose optimal strategies in subsequent moves.
- Sequential games differ from simultaneous games also by a simpler realization of *credible threats* - a situation when a player can make a move that would (in case of simultaneous game) decrease the payoff of his/her opponent, but also his/her own.
- In sequential game, the opponent can see that the player has already performed such action and he/she has to react to it.

EXCERCISE 4

A thug wants the contents of a safe and is threatening the owner, the only person who knows the code, to open the safe. "I will kill you if you dont open the safe, and let you live if you do." Should the information holder believe the threat and open the safe? The table shows the value that each person places on the various possible outcomes:

	Thug	Owner
Open the safe, thug does not kill	4	3
Open the safe, thug kills	2	1
Do not open, thug kills	1	2
Do not open, thug does not kill	3	4

Draw the game tree. Who moves first? What is the equilibrium? Does the safes owner believe the thugs threat?

In repeated games, players can make their move repeatedly during several periods

- Repeated games are such in which players repeat a simultaneous game during several periods.
- Players do not know what the others are doing in the given period, but they know what they did in previous periods.
- In repeated games, cooperative behavior can be achieved:
  - By repetition of a certain strategy, expected behavior can be signaled.
  - In subsequent periods, deviations from the cooperative strategy can be penalized.