Reducing Output Response Aliasing Using Boolean Optimization Techniques

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Abstract—In digital circuit testing, output response compaction can have a significant impact on fault coverage. The loss of fault coverage is caused by aliasing in the output response compaction. Classical approaches to reducing (eliminating) fault aliasing are based on modifications of the compactor design or modifying precomputed test sequence. In this paper, we propose a completely different approach based on a dedicated test pattern generation algorithm. The algorithm generates a test sequence with minimal aliasing for targeted faults. As the generated test sequence is tailored to given static and dynamic compactor structures, any response compactor can be used without a change in the design. We expand on our previous work, zero-aliasing ATPG, and incorporate pseudo-Boolean optimization techniques in the process.

The algorithm is evaluated using an LFSR-based MISR on a selection of benchmark circuits. A comparison with a state-of-the-art ATPG process without anti-aliasing measures is drawn.

Index Terms—ATPG, multiple-target test generation, SAT, pseudo-Boolean optimization, fault aliasing, response compaction, test compaction, LFSR

I. INTRODUCTION

The low test length and the low test application time are essential in both after-manufacture and in-deployment testing. The *Desing for testability* (DfT) techniques are used to simplify and speed-up test generation, test application, and to increase the fault coverage.

Output response compaction, a DfT technique, can be used to decrease the amount of read-out data during testing. This is useful both for *Built-In Self-Test* (BIST) and testing with external equipment. A *fault aliasing* can occur during the response compaction, decreasing the fault coverage. The output response compaction can be divided into *spatial* (static) and *temporal* (dynamic) parts.

Spatial response compaction is performed by a combinational circuit. It reduces the amount of data produced by a single test pattern, i.e., the output response bit width. Consecutive test patterns do not influence each other. The aliasing can be prevented in this compactor by its proper design [1]–[4].

Temporal response compaction reduces the number of response patterns and creates a signature of the test response. This is usually done by a *Multiple-Input Signature-Register* (MISR) – a sequential circuit that accumulates the output responses over multiple clock cycles in its internal state. One example of a MISR is a *Linear Feedback Shift Register* (LFSR) with parallel inputs.

Preventing aliasing in a temporal compactor is more complex because consecutive test patterns and output responses influence each other. Usual methods to prevent aliasing are manipulating the compactor design [5]–[9] or manipulating preexisting test pattern sequence [10].

A different approach to reducing the aliasing in the temporal response compaction is by constraining the ATPG algorithm to prevent aliasing in the compactor [11]. In this approach, the design of the compactor must be known during test generation. The ATPG then generates a test that is not aliased in the temporal (sequential) compactor. While this approach can achieve zero aliasing even for small and simple compactors, the trade-off is in the resulting test length and test generation time. Additionally, the generated test cannot be processed further, e.g., by a *static test compaction* [12]–[14].

Dynamic test compaction [15]–[20] is, in contrast to static compaction, performed during test pattern generation, on the incomplete test set. It is also beneficial for the ATPG to utilize extra information from the compaction while generating the next test pattern. Typical dynamic compaction tries to use the unspecified bits of generated test patterns to target additional faults. Other methods of dynamic compaction include *multiple-target test generation* (MTTG) [17]–[19] and *optimization-based MTTG* (OTG) [20] that can target multiple faults at once. Dynamic compaction, at the price of increased computational complexity. In this paper, we adopt modern dynamic compaction principles to the zero-aliasing test generation process. As a result, the test length is significantly reduced, compared to the original ZATPG algorithm.

In particular, the main contributions of this paper are as follows:

- Dynamic test compaction applied in zero-aliasing SATbased ATPG
- Optimization-based fault selection for anti-aliasing
- Evaluation and comparison with a state-of-the-art ATPG without anti-aliasing measures

The paper is structured as follows:

In Section II, there is a brief overview of recent work in the field as well as an introduction to tools being used. In Section III, the newly proposed method is described. The experimental



Figure 1: Miter: conceptual circuit for SAT-based ATPG

results are presented in section IV. Conclusions and discussed possibilities for future work are drawn in the last section V.

II. PRELIMINARIES

A. SAT-ATPG

SAT-based ATPG is an ATPG that uses the Boolean satisfiability problem (SAT) and a SAT solver to generate test patterns [21].

Construction of a *miter*, a conceptual circuit, is one of the possible ways to construct an SAT instance and generate a test pattern. Illustrated for the fault f_1 in Figure 1, two replicas of the circuit under test are created. One for the original fault-free circuit (CUT0), and the second for a faulty circuit (CUT1). The fault f_1 is modeled in the replica CUT1. Inputs (PIs) of the two replicas are connected and the outputs (POs) are compared. The output signal BD of the miter indicates whether a test pattern p on the PIs detects the fault f_1 or not. Finding a test pattern is then a problem of finding an input vector that sets the output of the circuit to the value 1.

This problem is then translated to SAT by means of the Tseitin transformation [21], [22]. The generated SAT instance is usually in the *conjunctive normal form* (CNF) as that is the input form that SAT solvers use.

When the SAT instance is satisfiable, then the certificate (Boolean assignment of the variables) corresponds to the values of signals in the miter. The test pattern is then extracted as the values of the input signals.

B. Multiple-Target Test Generation

Usual dynamic compaction techniques use unspecified bits of a test pattern to target additional faults. The compaction achieved by this approach is sensitive to the additional fault selection (fault ordering) and the chosen subset of unspecified bits in the test pattern.

Multiple-Target Test Generation (MTTG) techniques [17]–[19] solve the problem of unspecified bits selection by targeting multiple faults in one step. Modern SAT solvers are robust enough to efficiently find a test pattern for all selected faults or to prove that no such test pattern exists. This approach suffers from the need to efficiently find a subset of faults that can be tested by a single test pattern. In practice, faults are added to the targeted subset incrementally.

C. Optimization-Based MTTG

In the MTTG, a test pattern detecting all *selected* faults is found. If such a test pattern does not exist, the test pattern

is not generated, and a new fault set must be selected. *Optimization-based MTTG* (OTG) techniques [20] do not prove the non-existence of such test pattern. Instead, they compute a test pattern, which tests some subset of selected faults. The optimization criterion is the number of detected faults.

The algorithm presented in [20] works by constructing a miter for all selected faults. D-chains are also generated for each selected fault. The D-chains are not constrained to detect all faults, but their activation is done by the solver. The number of active D-chains, and thus detected faults, is maximized by the optimization function.

D. Reducing Aliasing in ATPG

The main idea behind the reduction of fault aliasing in an ATPG is constraining an SAT instance to not allow aliasing for selected faults (ZATPG) [11].

In this approach, the faults that were aliased by the previous (non-accepted) test pattern are selected for constraining (antialiasing). Constraining is done in the miter. In addition to generating a basic miter, as in Figure 1, the faults selected for anti-aliasing are appended as additional faulty replicas, similarly to the MTTG.

The aliasing happens in the temporal compactor, which is a sequential circuit. Aliasing can be detected by reading the internal compactor state only one clock cycle after the test pattern is applied to the inputs. As is illustrated in Figure 2, for the detection of aliasing during the SAT instance solving, the *unrolled temporal compactors*, i.e., their combinational parts, need to be appended to the outputs (POs) of each antialiasing replica. Outputs of these unrolled compactors are then constrained to differ from the fault-free compactor. The SAT solver is then forced to find a test pattern that would not alias these faults. The existence of such pattern is not guaranteed.

The requirement of zero aliasing for every test pattern is, however, too strong; instead, a relaxed constraint is used. The aliasing is allowed to occur, as long as the overall coverage is increased, i.e., the number of newly detected faults is higher than the number of aliased faults. If not, the aliased faults are added to the miter and the process repeats.

In short, the algorithm for generating a test pattern for a single fault is as follows. This procedure is performed for all faults in the fault list. First, a classical miter with the fault-free CUT (CUT0) and a selected fault f_1 (CUT1) is used to find a test pattern. Second, fault simulation is performed to identify the aliased faults. Third, if the test pattern does not increase the fault coverage, aliased faults (f_2, \ldots, f_n) are added to the miter and a test pattern is generated again. The resulting miter is illustrated in Figure 2.

If no test pattern can be found for the currently tested fault, it is skipped and visited later – it is moved to the end of the fault list. For more details on this basic ZATPG procedure, see [11].

This approach leads to high test generation times and the length of generated test is also not optimized. Further, the *static test compaction* cannot be used, because the aliasing



Figure 2: ZATPG: restricting fault aliasing

(and zero-aliasing property of the test) is dependent on the exact test pattern sequence and its ordering. Therefore, dy-namic test compaction must be used to reduce the number of test patterns. This was our main motivation for the proposed method described in the following section.

III. THE PROPOSED METHOD

In this section, we describe our approach to reduce *both* the aliasing and the test length. For that, we use an optimizing Pseudo-Boolean Optimization (PBO) solver in the ATPG process. The resulting proposed algorithm will be referenced as ZATPG-PBO.

A. Minimizing Fault Aliasing

In our approach, we build on our previous algorithm, ZATPG [11]. When using hard constraints on aliasing, we can get into a situation where no test pattern can be generated for our selection of anti-aliased faults (see Section II-D).

To avoid such a situation, we use an optimization criterion to minimize the number of aliased faults. Therefore, we let the solver select (some) minimal subset of anti-aliased faults to increase the probability that a test pattern for the currently selected fault exists.

Adding all faults to the miter for anti-aliasing would be too expensive – both in the miter construction and PBO solving time. Thus, we only add faults that are likely to be aliased. We detect such faults by simulation after a test pattern is generated. If necessary, we construct a new miter with appended faults and try to generate the test pattern again.

Example 1: Let us have a set of faults $F_a = \{f_1, \ldots, f_n\}$ that were aliased by a newly generated test pattern p_1 . The pattern is simulated and if the overall coverage has decreased because of aliasing, the pattern p_1 is not accepted. The aliased faults from F_a are added to the miter. A new test pattern p_2 is generated. The PBO solver minimizes the number of faults in F_a that are aliased by p_2 . The pattern p_2 is simulated and if the overall fault coverage increases, it is accepted.



Figure 3: Miter for multiple targeted and anti-aliased faults.

B. Test Compaction

Dynamic test compaction is used, because of the test's strict sequentiality – the aliasing depends on all test patterns and their ordering. For this reason, static test compaction or any other test post-processing cannot be used.

For this, we adapt recent OTG techniques [20]. With this method, we do not test a single fault; instead, we select a set of faults F_t to be tested. The faults are selected in the order of increasing *accidental detection index* ADI [23], i.e., faults that are less likely to be detected by an arbitrary pattern are selected first. Again, we use a PBO solver to select the maximal subset $F'_t \subset F_t$ of faults that can be tested by a single test pattern.

We combine this optimization criterion with the previous one, the aliasing minimization. Our goal is to maximize the overall fault coverage gained by the generated test pattern. The optimization cost of one newly detected fault is the same as the cost of one fault that was not aliased, i.e. we do not care if the pattern detects fault f_{t1} while aliasing fault f_{a1} or if neither is detected and aliased. This also leads to decreased complexity of the constructed miter – the newly selected faults and antialiased faults are uniform in the miter and PBO instance.

C. Miter and PBO Instance

The miter is constructed in a similar manner to the ZATPG algorithm (Figure 2), with the difference being that the faulty responses are not hard-constrained to be different from the fault-free response. Instead, they are included in the optimization criterion of the PBO instance. The optimization criterion is to maximize the number of differing faulty responses. See signals BD_1 through BD_n in Figure 3 and Equation 1.

$$\sum_{i=1}^{n} BD_i = max.$$
 (1)

The output of the solver is the test pattern with the maximum of selected faults detected (maximum of the BD_i signals set to the value 1). These are either newly targeted faults

or faults selected for anti-aliasing. Because the difference outputs are not constrained, this test pattern will always exist, possibly decreasing the fault coverage. The final change in fault coverage is known only after fault simulation.

D. The Algorithm Overview

Algorithm 1 General overview of the algorithm.						
1: procedure ATPG(C)						
2: $P \leftarrow \emptyset$						
3: $F \leftarrow faults(C)$						
4: repeat						
5: $F_t \leftarrow select(F)$						
6: $F_a \leftarrow \emptyset$						
7: repeat						
8: $p \leftarrow genPattern(C, F_t \cup F_a)$						
9: $F_d, F_a \leftarrow simulate(C, p)$						
10: until $ F_d - F_a \ge M_i \lor F_a \le M_a$						
11: if $ F_d - F_a \ge M_i$ then						
12: $P \leftarrow append(P, p)$						
13: $F \leftarrow update(F, F_d, F_a)$						
14: end if						
15: until $F = \emptyset \lor select(F) = \emptyset$						
16: return P						
17: end procedure						

A high-level view of the overall ZATPG-PBO algorithm is shown in Algorithm 1.

The algorithm is parametrized by the following variables: M_f default number of targeted faults

 M_{fm} maximum of targeted faults

 M_i acceptable improvement

 M_a the maximum of anti-aliased faults

The flow of the algorithm is as follows:

1) Fault Generation: On the line 3, a fault list for the circuit C is prepared. This fault list is precomputed and ordered by the ADI [23]. We also use this step to remove redundant faults from the fault-list.

2) Fault Selection: On line 5, a set F_t of targeted faults is selected from F. On the first invocation or after a test pattern was accepted, up to M_f first faults are selected. If a satisfactory test pattern was not found, new faults for F_t are selected (re-targeted [24]) and the first $n\frac{M_f}{2}$ faults are skipped for every (n) unsuccessful search.

3) Pattern Generation: On line 8, a test pattern p is generated, as described in previous subsections. The first run is done only for targeted faults F_t . Subsequent runs are done also for aliased faults F_a . This step is repeated until (line 10) a satisfactory pattern is generated or the number of aliased faults grows over the maximum number of anti-aliased faults M_a . Care must also be taken to detect that no satisfactory pattern exists. We do this by detecting that F_a was not changed since the last iteration.

4) Fault Simulation: On line 9, a fault simulation is done for all faults and the fault coverage is analyzed; aliased F_a and newly detected F_d faults are recorded. In addition to the circuit under test, the compactor is also simulated. The internal state of the compactor is kept for each fault by the algorithm.

5) Accepting Pattern: If the generated pattern is satisfactory – increases the fault coverage by at least M_i (line 11) – it is recorded (line 12). The internal state of the compactor is also updated for each fault and fault-free circuit. The fault list F is updated (line 13), detected faults are removed, and aliased faults are added back. The fault list remains ordered by ADI.

6) Algorithm Termination: The algorithm terminates when all faults are covered or if the fault list is exhausted (line 15). In the case of fault list exhaustion, there are some faults left. As the last attempt, the entire algorithm can be optionally restarted with increased number of targeted faults M_f . This restart keeps the already generated test set and the internal state of the algorithm. This is however effective only if the number of undetected faults is greater than M_f .

IV. EXPERIMENTAL RESULTS AND DISCUSSION

In this Section, the experimental results are presented. Achieved fault coverage for small-sized MISR is evaluated and compared to two ATPGs without anti-aliasing awareness, the Atalanta [25] ATPG and our implementation the OTG algorithm [24].

A. Experimental Setup

The ZATPG-PBO algorithm was implemented in the Go programming language. The used PBO solver is the Minisat+ [26]. The OTG algorithm was likewise re-implemented in the Go language, according to the algorithm described in [24]. Experiments were run on the circuits from ISCAS'85 [27], ISCAS'89 [28], ITC99 [29], and EPFL [30] benchmarks. For sequential circuits, their combinational profiles were extracted. The used fault model is the single stuck-at fault.

An output response compactor was appended to the primary outputs of the circuits in the following way: first, an irredundant spatial output response compactor was generated. The compactor was appended to the primary outputs of the CUT. The spatial compactor was constructed as a collection of disjoint elementary-gates trees [2], [3]. Second, an LFSRbased MISR with a primitive polynomial was used as the temporal output response compactor. The number of outputs of the spatial compactor was chosen to be equal to the size of the used MISR.

The experiments were run on the national grid infrastructure, consisting of heterogenous computation nodes. As such, computation times of the algorithm are only approximative.

From our pilot experiments, we have chosen the following setup: The maximal allowed *targeted test* window is set to $M_{fm} = 100$. Default *targeted test* window is set to the value $M_f = 50$. The acceptable improvement is set to $M_i = 40$. The allowed *anti-aliasing window* is set to $M_a = 100$.

B. Fault Coverage and Test Length

Fault coverage and test length for selected benchmark circuits are presented in Table I. Results for MISR sizes from 4 to 7 bits are selected, as the ZATPG-PBO generated a complete

Table I: Fault Coverage and Test Length

MISR SIZ	ZE		4						5					
algorithm			ZATPO	3-PBO	Ő	TG	Ata	lanta	ZATPO	3-PBO	Ő	TG	Ata	lanta
circuit	PO	faults	[%]	length	[%]	length	[%]	length	[%]	length	[%]	length	[%]	length
c499	32	990	83.33	14	95.35	52	96.16	53	92.42	30	97.07	52	98.18	53
c880	26	1754	99.89	23	94.93	19	93.04	59	100.00	22	97.32	18	97.09	56
c1355	32	2702	89.45	29	93.49	84	94.30	85	96.45	62	97.56	84	97.04	85
c1908	25	3805	90.17	41	93.43	110	92.88	120	96.08	66	96.19	108	96.85	120
c2670	140	4704			-				99.57	47	97.66	47	95.73	70
c7552	108	14253			-						-	-		
s832	24	1640	94.45	73	95.67	109	95.91	116	98.90	99	98.66	114	98.35	113
s1238	32	2310	92.81	60	94.72	123	94.46	128	95.67	69	97.45	125	96.84	130
b04	74	2553			-				99.80	40	97.06	38	95.93	67
b05	60	3671			-				99.16	47	97.03	48	96.49	84
b11	37	2227	97.80	51	94.57	54	94.79	73	99.55	50	96.32	50	97.75	68
b12	127	4615	94.19	60	95.47	97	93.24	149	99.37	89	97.55	98	97.68	150
cavlc	11	3052	96.76	125	93.64	162	94.00	136	98.10	124	97.38	157	96.43	136
dec	256	1840	54.24	14	96.30	256	95.65	256	49.62	12	98.04	256	97.93	256
i2c	142	5014			-				99.78	54	97.15	61	96.93	119
priority	8	2861	99.62	26	94.48	18	94.34	33	100.00	24	98.18	16	96.64	32
MISR SIZ	ZE		6						7					
MISR SIZ	ZE		6 ZATPO	G-PBO	O	TG	Ata	lanta	7 ZATPO	G-PBO	O	TG	Ata	lanta
MISR SIZ algorithm circuit	ZE PO	faults	6 ZATPO [%]	G-PBO length	0 [%]	TG length	Ata [%]	lanta length	7 ZATPO [%]	G-PBO length	0 [%]	TG length	Ata [%]	lanta length
MISR SIZ algorithm circuit c499	ZE PO 32	faults 990	6 ZATPO [%] 99.90	G-PBO length 56	0 [%] 98.38	TG length 52	Ata [%] 98.48	lanta length 53	7 ZATPO [%] 100.00	G-PBO length 54	0' [%] 99.70	TG length 52	Ata [%] 99.39	lanta length 53
MISR SIZ algorithm circuit c499 c880	ZE PO 32 26	faults 990 1754	6 ZATPO [%] 99.90 100.00	G-PBO length 56 24	0 [%] 98.38 99.09	TG length 52 20	Ata [%] 98.48 98.86	lanta length 53 58	7 ZATPO [%] 100.00 100.00	G-PBO length 54 20	0 [%] 99.70 99.03	TG length 52 18	Ata [%] 99.39 99.20	lanta length 53 59
MISR SIZ algorithm circuit c499 c880 c1355	ZE PO 32 26 32	faults 990 1754 2702	6 ZATPO [%] 99.90 100.00 100.00	G-PBO length 56 24 88	0' [%] 98.38 99.09 98.41	TG length 52 20 84	Ata [%] 98.48 98.86 98.67	lanta length 53 58 85	7 ZATPO [%] 100.00 100.00 100.00	G-PBO length 54 20 86	07 [%] 99.70 99.03 98.93	TG length 52 18 85	Ata [%] 99.39 99.20 99.48	lanta length 53 59 85
MISR SIZ algorithm circuit c499 c880 c1355 c1908	ZE PO 32 26 32 25	faults 990 1754 2702 3805	6 ZATPO [%] 99.90 100.00 100.00 96.48	G-PBO length 56 24 88 63	0' [%] 98.38 99.09 98.41 98.63	TG length 52 20 84 110	Ata [%] 98.48 98.86 98.67 99.00	lanta length 53 58 85 117	7 ZATPO [%] 100.00 100.00 100.00 99.97	G-PBO length 54 20 86 109	07 [%] 99.70 99.03 98.93 99.50	TG length 52 18 85 109	Ata [%] 99.39 99.20 99.48 99.66	lanta length 53 59 85 118
MISR SIZ algorithm circuit c499 c880 c1355 c1908 c2670	ZE PO 32 26 32 25 140	faults 990 1754 2702 3805 4704	6 ZATPO [%] 99.90 100.00 100.00 96.48 100.00	G-PBO length 56 24 88 63 56	0' [%] 98.38 99.09 98.41 98.63 98.94	TG length 52 20 84 110 48	Ata [%] 98.48 98.86 98.67 99.00 96.19	lanta length 53 58 85 117 70	7 ZATPO [%] 100.00 100.00 99.97 99.74	G-PBO length 54 20 86 109 41	0' [%] 99.70 99.03 98.93 99.50 99.36	TG length 52 18 85 109 45	Ata [%] 99.39 99.20 99.48 99.66 97.53	lanta length 53 59 85 118 71
MISR SIZ algorithm circuit c499 c880 c1355 c1908 c2670 c7552	ZE PO 32 26 32 25 140 108	faults 990 1754 2702 3805 4704 14253	6 ZATPO [%] 99.90 100.00 100.00 96.48 100.00 99.90	G-PBO length 56 24 88 63 56 55	0' [%] 98.38 99.09 98.41 98.63 98.94 98.79	TG length 52 20 84 110 48 48	Ata [%] 98.48 98.67 99.00 96.19 98.61	lanta length 53 58 85 117 70 151	7 ZATPO [%] 100.00 100.00 99.97 99.74 100.00	G-PBO length 54 20 86 109 41 50	0' [%] 99.70 99.03 98.93 99.50 99.36 99.22	TG length 52 18 85 109 45 47	Ata [%] 99.39 99.20 99.48 99.66 97.53 99.23	lanta length 53 59 85 118 71 139
MISR SIZ algorithm circuit c499 c880 c1355 c1908 c2670 c7552 s832	ZE PO 32 26 32 25 140 108 24	faults 990 1754 2702 3805 4704 14253 1640	6 ZATPO [%] 99.90 100.00 100.00 96.48 100.00 99.90 99.27	G-PBO length 56 24 88 63 56 55 105	0' [%] 98.38 99.09 98.41 98.63 98.94 98.79 99.39	TG length 52 20 84 110 48 48 109	Ata [%] 98.48 98.86 98.67 99.00 96.19 98.61 99.45	lanta length 53 58 85 117 70 151 114	7 ZATPO [%] 100.00 100.00 99.97 99.74 100.00 100.00	G-PBO length 54 20 86 109 41 50 108	07 [%] 99.70 99.03 98.93 99.50 99.36 99.22 99.39	TG length 52 18 85 109 45 47 111	Ata [%] 99.39 99.20 99.48 99.66 97.53 99.23 99.51	lanta length 53 59 85 118 71 139 114
MISR SIZ algorithm circuit c499 c880 c1355 c1908 c2670 c7552 s832 s1238	ZE PO 32 26 32 25 140 108 24 32	faults 990 1754 2702 3805 4704 14253 1640 2310	6 ZATPO [%] 99.90 100.00 96.48 100.00 99.90 99.27 96.75	G-PBO length 56 24 88 63 56 55 105 70	0' [%] 98.38 99.09 98.41 98.63 98.94 98.79 99.39 99.22	TG length 52 20 84 110 48 48 48 109 123	Ata [%] 98.48 98.86 98.67 99.00 96.19 98.61 99.45 98.92	lanta length 53 58 85 117 70 151 114 127	7 ZATPO [%] 100.00 100.00 99.97 99.74 100.00 100.00 99.78	G-PBO length 54 20 86 109 41 50 108 113	07 [%] 99.70 99.03 98.93 99.50 99.36 99.22 99.39 99.26	TG length 52 18 85 109 45 47 111 128	Ata [%] 99.39 99.20 99.48 99.66 97.53 99.23 99.51 98.92	lanta length 53 59 85 118 71 139 114 130
MISR SIZ algorithm circuit c499 c880 c1355 c1908 c2670 c7552 s832 s1238 b04	ZE PO 32 26 32 25 140 108 24 32 74	faults 990 1754 2702 3805 4704 14253 1640 2310 2553	6 ZATPO [%] 99.90 100.00 96.48 100.00 99.90 99.27 96.75 99.96	G-PBO length 56 24 88 63 56 55 105 70 42	0 [%] 98.38 99.09 98.41 98.63 98.94 98.79 99.39 99.22 98.94	TG length 52 20 84 110 48 48 48 109 123 39	Ata [%] 98.48 98.86 98.67 99.00 96.19 98.61 99.45 98.92 98.90	lanta length 53 58 85 117 70 151 114 127 66	7 ZATPO [%] 100.00 100.00 99.97 99.74 100.00 100.00 99.78 100.00	G-PBO length 54 20 86 109 41 50 108 113 39	0 [%] 99.70 99.03 98.93 99.50 99.36 99.22 99.39 99.26 99.29	TG length 52 18 85 109 45 47 111 128 39	Ata [%] 99.39 99.20 99.48 99.66 97.53 99.23 99.51 98.92 99.29	lanta length 53 59 85 118 71 139 114 130 67
MISR SIZ algorithm circuit c499 c880 c1355 c1908 c2670 c7552 s832 s1238 b04 b05	ZE <u>PO</u> 32 26 32 25 140 108 24 32 74 60	faults 990 1754 2702 3805 4704 14253 1640 2310 2553 3671	6 ZATPO [%] 99.90 100.00 96.48 100.00 99.90 99.27 96.75 99.96 100.00	G-PBO length 56 24 88 63 56 55 105 70 42 53	0 [%] 98.38 99.09 98.41 98.63 98.94 98.79 99.39 99.22 98.94 99.18	TG length 52 20 84 110 48 48 109 123 39 51	Ata [%] 98.86 98.86 98.67 99.00 96.19 98.61 99.45 98.92 98.90 98.72	lanta length 53 58 85 117 70 151 114 127 66 84	7 ZATPO [%] 100.00 100.00 99.97 99.74 100.00 100.00 99.78 100.00 100.00	G-PBO length 54 20 86 109 41 50 108 113 39 49	0 [%] 99.70 99.03 98.93 99.50 99.36 99.22 99.39 99.26 99.29 98.86	TG length 52 18 85 109 45 47 111 128 39 48	Ata [%] 99.39 99.20 99.48 99.66 97.53 99.23 99.23 99.51 98.92 99.29 99.54	lanta length 53 59 85 118 71 139 114 130 67 83
MISR SIZ algorithm circuit c499 c880 c1355 c1908 c2670 c7552 s832 s1238 b04 b05 b11	ZE <u>PO</u> 32 26 32 25 140 108 24 32 74 60 37	faults 990 1754 2702 3805 4704 14253 1640 2310 2553 3671 2227	6 ZATPO [%] 99.90 100.00 96.48 100.00 99.90 99.27 96.75 99.96 100.00 99.91	G-PBO length 56 24 88 63 56 55 105 70 42 53 57	00 [%] 98.38 99.09 98.41 98.63 98.94 99.39 99.39 99.22 98.94 99.18 99.18 98.83	TG length 52 20 84 110 48 48 48 109 123 39 51 53	Ata [%] 98.48 98.86 98.67 99.00 96.19 98.61 99.45 98.92 98.90 98.72 98.25	lanta length 53 58 85 117 70 151 114 127 66 84 69	7 ZATPO [%] 100.00 100.00 99.97 99.74 100.00 100.00 99.78 100.00 99.96	G-PBO length 54 20 86 109 41 50 108 113 39 49 50	0 [%] 99.70 99.03 99.50 99.36 99.22 99.39 99.26 99.29 98.86 99.25	TG length 52 18 85 109 45 47 111 128 39 48 53	Ata [%] 99.39 99.20 99.48 99.66 97.53 99.23 99.51 99.52 99.29 99.54 98.92	lanta length 53 59 85 118 71 139 114 130 67 83 67
MISR SIZ algorithm circuit c499 c880 c1355 c1908 c2670 c7552 s832 s1238 b04 b05 b11 b12	ZE PO 32 26 32 25 140 108 24 32 74 60 37 127	faults 990 1754 2702 3805 4704 14253 1640 2310 2553 3671 2227 4615	6 ZATPO [%] 99.90 100.00 96.48 100.00 99.90 99.27 96.75 99.96 100.00 99.91 99.85	G-PBO length 56 24 88 63 56 55 105 70 42 53 57 88	0 [%] 98.38 99.09 98.41 98.63 98.94 99.39 99.22 98.94 99.18 99.18 99.83 98.83 98.44	TG length 52 20 84 110 48 48 109 123 39 51 53 91	Ata [%] 98.48 98.86 98.67 99.00 96.19 98.61 99.45 98.92 98.90 98.72 98.25 97.83	lanta length 53 58 85 117 70 151 114 127 66 84 69 145	7 ZATPO [%] 100.00 100.00 99.97 99.74 100.00 100.00 99.78 100.00 99.96 100.00	G-PBO length 54 20 86 109 41 50 108 113 39 49 50 92	0 [%] 99.70 99.03 99.50 99.36 99.22 99.39 99.26 99.29 98.86 99.55 99.68	TG length 52 18 85 109 45 47 111 128 39 48 53 96	Ata [%] 99.39 99.20 99.48 99.66 97.53 99.23 99.51 98.92 99.29 99.54 98.92 99.54	lanta length 53 59 85 118 71 139 114 130 67 83 67 145
MISR SIZ algorithm circuit c499 c880 c1355 c1908 c2670 c7552 s832 s1238 b04 b05 b11 b12 cavlc	ZE PO 32 26 32 25 140 108 24 32 74 60 37 127 11	faults 990 1754 2702 3805 4704 14253 1640 2310 2553 3671 2227 4615 3052	6 ZATPC [%] 99.90 100.00 96.48 100.00 99.90 99.27 96.75 99.96 100.00 99.91 99.85 99.15	G-PBO length 56 24 88 63 56 55 105 70 42 53 57 88 135	07 [%] 98.38 99.09 98.41 98.63 98.94 98.79 99.39 99.22 98.94 99.18 98.83 98.83 98.84 98.83	TG length 52 20 84 110 48 48 109 123 39 51 53 91 156	Ata [%] 98.48 98.86 98.67 99.00 96.19 98.61 99.45 98.92 98.90 98.72 98.72 98.25 97.83 97.97	lanta length 53 58 85 117 70 151 114 127 66 84 69 145 136	7 ZATPC [%] 100.00 100.00 99.97 99.74 100.00 100.00 99.78 100.00 99.96 100.00 99.64	G-PBO length 54 20 86 109 41 50 108 113 39 49 50 92 142	0 [%] 99.70 99.03 99.50 99.36 99.22 99.39 99.26 99.29 98.86 99.55 99.68 99.57	TG length 52 18 85 109 45 47 111 128 39 48 53 96 156	Ata [%] 99.39 99.20 99.48 99.66 97.53 99.23 99.51 98.92 99.29 99.54 98.92 99.54 98.92	lanta length 53 59 85 118 71 139 114 130 67 83 867 145 136
MISR SIZ algorithm circuit c499 c880 c1355 c1908 c2670 c7552 s832 s1238 b04 b05 b11 b12 cavlc dec	ZE PO 32 26 32 25 140 108 24 32 74 60 37 127 11 256	faults 990 1754 2702 3805 4704 14253 1640 2310 2553 3671 2227 4615 3052 1840	6 ZATPC [%] 99.90 100.00 90.00 99.90 99.27 96.75 99.96 100.00 99.91 99.85 99.15 68.21	G-PBO length 56 24 88 63 56 55 105 70 42 53 57 88 135 62	07 [%] 98.38 99.09 98.41 98.63 98.94 98.79 99.39 99.22 98.94 99.18 98.83 98.44 98.83 98.44 98.92 99.24	TG length 52 20 84 110 48 48 109 123 39 51 53 91 156 256	Ata [%] 98.48 98.86 98.67 99.00 96.19 98.61 99.45 98.92 98.92 98.92 98.72 98.72 98.72 98.72 98.72 98.72	lanta length 53 58 85 117 70 151 114 127 66 84 69 145 136 256	7 ZATPO [%] 100.00 100.00 99.97 99.74 100.00 100.00 99.78 100.00 99.96 100.00 99.64 78.75	G-PBO length 54 20 86 109 41 50 108 113 39 49 50 92 142 126	0 [%] 99.70 99.03 99.50 99.36 99.22 99.39 99.26 99.29 98.86 99.55 99.68 99.57 99.46	TG length 52 18 85 109 45 47 111 128 39 48 53 96 156 256	Ata [%] 99.39 99.20 99.48 99.66 97.53 99.23 99.51 98.92 99.54 98.92 99.54 98.92 99.54 98.92 99.54 98.69 99.08	lanta length 53 59 85 118 71 139 114 130 67 83 3 67 145 136 256
MISR SIZ algorithm circuit c499 c880 c1355 c1908 c2670 c7552 s832 s1238 b04 b05 b11 b12 cavlc dec i2c	ZE PO 32 26 32 25 140 108 24 32 74 60 37 127 11 256 142	faults 990 1754 2702 3805 4704 14253 1640 2310 2553 3671 2227 4615 3052 1840 5014	6 ZATPO [%] 99.90 100.00 96.48 100.00 99.90 99.27 96.75 99.96 100.00 99.91 99.85 99.15 68.21 99.84	G-PBO length 56 24 88 63 56 55 105 70 42 53 57 88 135 62 52	07 [%] 98.38 99.09 98.41 98.63 98.64 98.79 99.39 99.22 98.94 99.18 98.83 98.83 98.83 98.44 98.92 99.24 99.04	TG length 52 20 84 110 48 48 48 109 123 39 51 53 91 156 256 57	Ata [%] 98.48 98.86 98.67 99.00 96.19 98.61 99.45 98.92 98.92 98.92 98.72 98.25 97.83 97.97 99.24 97.37	lanta length 53 58 85 117 70 151 114 127 66 84 69 145 136 256 117	7 ZATPO [%] 100.00 100.00 99.97 99.74 100.00 99.78 100.00 99.96 100.00 99.64 78.75 99.92	G-PBO length 54 20 86 109 41 50 108 113 39 49 50 92 142 126 55	0 [%] 99.70 99.03 99.50 99.50 99.22 99.39 99.26 99.29 98.86 99.55 99.88 99.55 99.57 99.46 99.48	TG length 52 18 85 109 45 47 111 128 39 48 53 96 156 256 63	Ata [%] 99.39 99.20 99.48 99.66 97.53 99.23 99.51 98.92 99.54 98.92 99.54 98.92 99.54 98.92 99.46 98.69 99.08 98.62	lanta length 53 59 85 118 71 139 114 130 67 83 67 145 136 256 116

test in this range for most circuits. Missing values indicate that the computation did not terminate within the time limit or irredundant static compactor was not available.

The results are compared with Atalanta ATPG [25], which uses static test compaction, and with the OTG [24], which uses the same dynamic test compaction as the ZATPG-PBO.

Our algorithm produces tests of similar length as the OTG while achieving higher fault coverage in most cases, especially for bigger compactors. The OTG and Atalanta produce fault coverage consistent with the theoretical probability of fault masking in an LFSR (2^{-n}) .

For some circuits, the ZATPG-PBO performs poorly for small compactor sizes. This can be explained by early termination in the case when no test pattern capable of improving fault coverage is found for any remaining fault. Of note is the circuit *dec*, where ZATPG-PBO performs very poorly; full coverage is achieved for MISR of size 8 bits with a test length of 258 patterns. Note that both OTG and Atalanta generated exhaustive tests – 256 patterns for 8-bit PI.

C. Computation time

Table II shows computation times. The computation time increases for smaller compactors, where the aliasing probabil-

Table II: Computation time

MISR	3	4	5	6	7	8	9
circuit	[s]	[s]	[s]	[s]	[s]	[s]	[s]
c499	149694	4398	3713	1623	1497	747	564
c880	239315	75849	52237	74553	40264	18681	13332
c1355	74617	15462	19391	10894	4052	2427	1701
c1908	237705	98428	47796	19248	9143	5209	3776
c2670	-	-	71337	31625	10699	5732	3787
c7552	-	-	-	89363	52530	57445	5712
s832	-	1567	1139	985	953	901	813
s1238	55594	31575	13013	5015	3089	2425	1736
b04	-	-	304154	19479	12266	4369	2082
b05	-	208634	212417	84267	14621	7401	5516
b11	41364	34454	18744	12873	5656	2722	2778
b12	-	115641	38641	14748	8751	6479	3654
cavlc	8200	9863	5513	3443	2985	3459	3167
dec	647	1622	831	5222	8406	10342	7600
i2c	-	-	254353	10211	7395	2391	1823
max	-	-	-	-	271157	101279	70130
priority	5919	6299	496	191	186	186	-

ity is higher. This leads to bigger miters and PBO instances and also to a higher number of PBO solver invocations.

For some circuits, there is a visible step in computation time that coincides with runs where the number of anti-aliased faults reached limit $M_a = 100$. This also indicates that fault re-targeting is not as efficient as fault anti-aliasing.

V. CONCLUSIONS AND FUTURE WORK

The aim of this work was to propose a test generation process minimizing aliasing in the temporal response compactor, *without the need to modify its structure*. This is in contrast to other approaches that aim at the compactor redesign. As a result, satisfactory fault coverage can be achieved, even for small compactors, without sacrificing the test length. Therefore, the BIST area can be reduced "for free"; all the effort is moved to the algorithmic level.

The presented method is a modified ATPG process based on a Pseudo-Boolean Optimization problem, ZATPG-PBO. This algorithm is an improvement upon our previous algorithm, ZATPG [11].

The algorithm is evaluated in terms of fault coverage, test length, and computation time. Our algorithm achieves better fault coverage than ATPG without fault anti-aliasing. The test length is comparable to a state-of-the-art ATPG with modern dynamic test compaction.

The algorithm was tested with the LFSR temporal compactor but can work with any compactor for which the combinational logic can be extracted.

As a future work, we propose evaluating different temporal compactors, for example, cellular automata or non-linear compactors. Additionally, the influence of all parameters and their interaction is not completely understood.

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