Václav Blažej
vaclav.blazej@fit.cvut.cz
Presented paper by Lucia Moura, Sebastian Raaphorst and Brett Stevents

The Lovász Local Lemma and Variable Strength Covering Arrays

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Introduction

Lovász Local Lemma is a probabilistic technique to get an existence proof in settings where events are only sparsely dependent. We will use this lemma to get an upper bound on variable strenght covering array (VCA). We will compare this bound with known bounds on several classes of hypergraphs.

Definition 1 (VCA & VCAN) *Let $H = (V, E)$ be a hypergraph and let $k = |V|$. A* variable strength covering array*, denoted $VCA(n; H, v)$, is an $n \times k$ array M filled from $\mathbb{Z}_v$ such that for $e = \{v_0, \dots, v_{t-1}\} \in E$, the $n \times t$ subarray of columns indexed by e is* covered*, that is it has every possible t-tuple in $\mathbb{Z}_v$ as a row at least once. The* variable-strength covering array number*, written $VCAN(H, v)$, is the smallest n such that a $VCA(n; H, v)$ exists.*

Theorem 2 (Lovász local lemma - Symmetric Case) *Consider a finite set of events $\mathcal{A} = \{A_0, \dots, A_{m-1}\}$ in a probability space Ω such that each event occurs with probability at most $p < 1$, and each event is independent of all but at most d of the other events. If $ep(d + 1) \leq 1$, where e is the base of the natural logarithm, then the probability that none of the events occur is nonzero.*

Main result

Theorem 3 *Let $H = (V, E)$ be a hypergraph with $rank(H) = t \geq 1$, and let d be an integer such that no edge of H intersects more than d other edges of H. Then, for any $v \geq 2$, we have:*

$$VCAN(H, v) \leq \left\lceil \frac{\ln(d + 1) + t \ln v + 1}{\ln \frac{v^t}{v^t - 1}} \right\rceil \tag{1}$$

Definition 4 *A* design *is a pair $(X, \mathcal{A})$ such that the following properties are satisfied: X is a set of elements called* points*, and $\mathcal{A}$ is a collection (i.e., multiset) of nonempty subsets of X called* blocks*.*

Definition 5 *Let $v, k,$ and λ be a positive integers such that $v > k \geq 2$. A t-(v, k, λ)* balanced incomplete block design (BIBD) *is a design $(X, \mathcal{A})$ such that the following properties are satisfied: $|X| = v$, each block contains exactly k points, each t-element subset of X is contained in exactly λ blocks.*

Sometimes we use notation (v, b, r, k, λ) to emphasize number of blocks r and in how many blocks each point occurs r. These two parameters depend only on v, k, λ.

Definition 6 *The* cyclic consecutive hupergraph *is $H_c^{k,t} = (V, E)$ with $V = \{0, \dots, k - 1\}$ and $E = \{\{i, i + 1 \bmod k, \dots, i + t - 1 \bmod k\} : 0 \leq i \leq k - 1\}$*

Definition 7 *A* triangulation hypergraph of the sphere*, $T = (V, E)$ is a rank-3 hypergraph which corresponds to a planar graph all of whose faces are triangles; the rank-3 hyperedges are precisely the faces of the planar embedding.*