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presented:

Graph editing to a fixed target

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Definitions

- Edge contraction is removal of edge $e = (u, v)$ and identification of vertices u, v.
- Vertex dissolution is the removal of a vertex v with exactly two neighbors u and w, which may not be adjacent to each other, followed by the inclusion of the edge uw.
- Relation is induced if edge deletions are excluded from the permitted graph operations.
- A graph H is called a topological minor of a graph G if a subdivision of H is isomorphic to a subgraph of G.

Problems

H-MINOR EDIT PROBLEM	
Input:	Graph G and an integer k
Decide:	Can G be modified into H by at most k operations?
Operations:	Edge contraction, edge deletion, vertex deletion
H-TOPOLOGICAL MINOR EDIT PROBLEM	
Input:	Graph G and an integer k
Decide:	Can G be modified into H by at most k operations?
Operations:	Vertex disolution, edge deletion, vertex deletion

Theorems

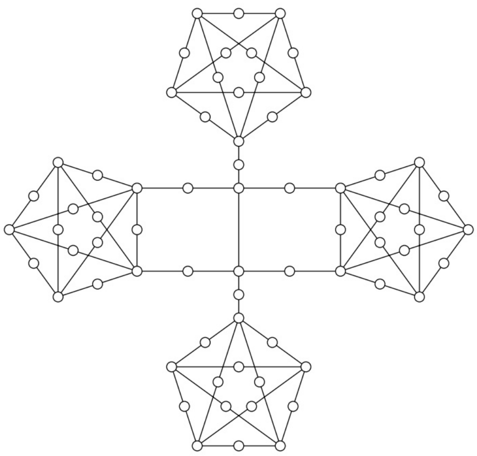

Figure 1: The smallest graph H^* for which H-INDUCED MINOR is *NP-complete*

Lemma 1. H-MINOR can be solved in cubic time for all graphs H.
Lemma 2. H^*-INDUCED MINOR is *NP-complete.*
Lemma 3. K_5-INDUCED TOPOLOGICAL MINOR is *NP-complete.*
Lemma 4. If (G, k) is a yes-instance of H-MINOR EDIT or H-TOPOLOGICAL MINOR EDIT, for some graph H, then $|VH| \leq |VG| \leq |VH| + k$.
Lemma 5. Let H be a graph. Then the following two statements hold:

1. H-INDUCED MINOR $\leq$ H-MINOR EDIT.
2. H-INDUCED TOPOLOGICAL MINOR $\leq$ H-TOPOLOGICAL MINOR EDIT.

Lemma 6. Let H be a graph and k an integer. If a graph G has an H-minor sequence of length k, then G has a *nice* H-minor sequence of length at most k.

Lemma 7. Let H be a graph and k an integer. If a graph G has an H-topological minor sequence of length k, then G has a *seminice* H-topological minor sequence S of length at most k, such that the vertices not deleted by the vertex deletions of S induce a subgraph that contains a subdivision of H as a spanning subgraph.

Lemma 8. Let $\mathcal{G}$ be any nontrivial minor-closed graph class. Then, for all graphs H, the H-INDUCED MINOR problem can be solved in linear time on $\mathcal{G}$.

Lemma 9. For all graphs H, the H-INDUCED MINOR problem can be solved in polynomial time on AT-free graphs.

Lemma 10. For all graphs H, the H-INDUCED MINOR problem can be solved in polynomial time on chordal graphs

Lemma 11. Let $\mathcal{G}$ be a graph class and H a graph. If H'-INDUCED MINOR is polynomial-time solvable on $\mathcal{G}$ for each spanning supergraph H' of H, then H-MINOR EDIT is polynomial-time solvable on $\mathcal{G}$.

Theorem 1. The following two statements hold:

1. There is a graph H for which H-MINOR EDIT is *NP-complete*.
2. There is a graph H for which H-TOPOLOGICAL MINOR EDIT is *NP-complete*.

Theorem 2. The following two statements hold:

1. K_r-MINOR EDIT can be solved in cubic time for all $r \geq 1$.
2. K_r-TOPOLOGICAL MINOR EDIT can be solved in polynomial time, if $r \leq 3$, and is *NP-complete*, if $r \geq 5$.

Theorem 3. P_r-MINOR EDIT and $K_{1,r}$-MINOR EDIT can both be solved in polynomial time for all $r \geq 1$.

Theorem 4. Let H be a subdivided star. Then H-TOPOLOGICAL MINOR EDIT is polynomial-time solvable.

Theorem 5. C_r-TOPOLOGICAL MINOR EDIT can be solved in polynomial time for all $r \geq 3$.

Theorem 6. For all graphs H, the H-MINOR EDIT problem is polynomial-time solvable on

1. the class of AT-free graphs,
2. the class of chordal graphs,
3. any nontrivial minor-closed class of graphs.