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presented:

Fast Polynomial-Space Algorithms Using Inclusion-Exclusion

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https://link.springer.com/article/10.1007/s00453-012-9630-x

Theorem 1. (Inclusion-Exclusion) Let U and R be sets and for every $v \in R$, let P_v be a subset of U. Use $\overline{P_v}$ to denote $U\ P_v$. With the convention $\bigcap_{i \in \emptyset} \overline{P_i} = U$, the following holds:

$$\left| \bigcap_{v \in R} P_v \right| = \sum_{F \subseteq R} (-1)^{|F|} \left| \bigcap_{v \in F} \overline{P_v} \right|$$

We refer to the set U as the *universe*, to R as the *requirement space* and to P_v as a *property*.

Problems to Solve

#HAMILTONIAN PATHS

Input $G = (V, E)$

Question The number of Hamiltonian paths in G.

STEINER TREE

Input $G = (V, E), c \in \mathbb{Z}^+$, weight function $w : E \to [c] \setminus 0$ and terminals $K \subseteq V$.

Question Is there a subtree (V', E') of G such that $K \subset V'$ and $\sum_{e \in E'} w(e) \leq c$?

DEGREE CONSTRAINED SPANNING TREE

Input $G = (V, E), 1 \leq c \leq n$.

Question Is there a spanning tree of G with maximum degree at most c?

MAXIMUM INTERNAL SPANNING TREE

Input $G = (V, E), 1 \leq c \leq n$.

Question Is there a spanning tree of G with at least c internal vertices?

#SPANNING FORESTS

Input $G = (V, E), 1 \leq c \leq n$.

Question The number of acyclic spanning subgraphs of G with exactly c connected components.