Patrik Nikl

niklpatr@fit.cvut.cz

presented:

Induced colorful trees and paths in large chromatic graphs

András Gyárfás, Gábor N. Sárközy,

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Definitions

- In a proper vertex coloring of a graph a subgraph is colorful if it's vertices are colored with different colors
- In graph theory, an induced subgraph of a graph is another graph, formed from a subset of the vertices of the graph and all of the edges connecting pairs of vertices in that subset.
- Assume we have a proper coloring on G. The color degree $cod_G(v)$ is the number of distinct colors appearing on the neighbors of v and $cod(G) = max\{cod_G(v) : v \in V(G)\}$.

Theorems

Conjecture 1. In any proper coloring of any triangle free k-cromatic graph G there is an induced colorful path on k vertices.

Theorem 1. Let T_k be a tree on k vertices. Then every $\{C_3, C_4\}$-free graph of minimum degree at least $k - 1$ contains T_k as an induced subgraph.

Theorem 2. Let T_k be a tree on $k >= 4$ vertices. Then every proper coloring of $\{C_3, C_4\}$-free graph G with $cod(G) >= 2k - 5$ contains T_k as an induced colorful subgraph.

Theorem 3. Let k be a positive integer and T_k be a tree on k vertices. There exists a function $f(k)$ such that the following holds. If G is a $\{C_3, C_4\}$-free graph with $\chi(G) >= f(k)$ then in any proper coloring of G and in any acyclic orientation of G there is either an induced colorful T_k or an induced directed path P_k.

Lemma 1. $P = P_t$ contains an induced P_k starting from the first vertex of P.