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presented:

Solving the train marshalling problem by inclusion-exclusion

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Definitions

- Given a sequence α of length n and a subset $S \subseteq N_n = \{1, \dots, n\}$ we denote by $\alpha[S]$ the sequence obtained from α by removing the elements in positions in $N_n \setminus S$. If $S = \{i\}$, we also write $\alpha[S] = \alpha[i] = \alpha_i$. Given two sequences α and β of length m and n, respectively, we denote by $\alpha \cdot \beta$ the concatenation of α and β, that is the sequence $(\alpha_1, \dots, \alpha_m, \beta_1, \dots, \beta_n)$. Sequence $\sigma(n, k) = (1, 2, \dots, n, 1, 2, \dots, n, \dots, 1, 2, \dots, n)$ is obtained by replicating k times the sequence (1,2,...,n). Finally, any (total) order of set N_n is represented by the sequence τ of length n where $\tau[i]$, $i = 1, \dots, n$, denotes the i-th element in the order.
- An instance of the TMP is a triple (n, t, D) where n is the number of cars of the train, t is the number of destinations and D is a partition of N_n in subsets $D(j)$, $j \in N_t$. Each set $D(j)$ contains the indices of the cars having destination j. We will identify the cars of the train simply with their index. In this way the original order of the cars corresponds to the sequence $(1, 2, \dots, n)$.
- An order τ of N_n is said a TM-order for the instance (n, t, D) if the elements of each set $D(j)$, $j \in N_t$, appear consecutively in τ, i.e., $\tau[r]$, $\tau[s] \in D(j)$ for some $1 \le r < s \le n$ implies $\tau[i] \in D(j)$ for every $r \le i \le s$.
- An instance of the TMP can be reordered by means of k auxiliary tracks to obtain a TM-train if and only if there exists a map $\phi : N_n \to N_k$ such that setting $\phi^{-1}(r) = \{i \in N_n \mid \phi(i) = r\}$ for each $r \in N_k$, order $\tau^\phi = \alpha[\phi^{-1}(1)] \cdot \alpha[\phi^{-1}(2)] \cdots \alpha[\phi^{-1}(k)]$ is a TM-order of N_n. The map ϕ is in this case called k-TM-solution or briefly k-solution of the TMP instance.
- The sequence $\sigma(n, k)$ covers the partition D if there exists a subset $R \subseteq N_{nk}$, $|R| = n$, with the property that $\sigma(n, k)[R]$ is a TM-order of N_n. In this case we say that $\sigma(n, k)$ covers D according to R.
- Given a TMP instance (n, t, D) and $k \in N$ directed graph $G(n, t, D, k) = (V, E)$ is defined as follows. Node set $V = \{1, 2, \dots, nk\} \cup \{0, nk + 1\}$ contains node for each position in sequence $\sigma = \sigma(n, k)$ and two auxiliary nodes 0 and $nk + 1$. Arc set $E = \widehat{E} \cup E_{nk+1}$ where $\widehat{E} = \{(i, \psi(i, j) \mid i \in \{0, 1, \dots, nk\}, j \in N_t, \psi(i, j) \neq nk + 1\}$, function $\psi : \{0, 1, \dots, nk\} \times N_t \to N_{nk}$ is defined as $\psi(i, j) = \min(\{l > i \mid D(j) \subseteq (N_l \setminus N_i)\} \cup \{nk + 1\})$, and $E_{nk+1} = \{(i, nk + 1) \mid i \in V \setminus \{nk + 1\}\}$.
- Arcs of the graph $G(n, t, d, k)$ are colored with $t + 1$ different colors by assigning color j to every arc $(i, j) \in \widehat{E}$ such that $\sigma[h] \in D(j)$ and by assigning color $t + 1$ to each arc of E_{nk+1}. $C[e]$ denotes color of arc $e \in E$ and we call j-arc any arc of color j.
- Let $J \subseteq N_t$. A J-rainbow path in the graph $G(n, t, D, k)$ is a directed path P with $|J|$ arcs that contains (exactly) one arc of color j for each $j \in J$.
- Contracted instance $C(n, t, D)$ of (n, t, D) is the instance obtained by recursively removing a car from the train if the following car has the same destination.

Problems

Train Marshalling Problem (TMR)
Given a TMP instance (n, t, D) find the minimum $k \in N$ such that there exists a k-solution.
Decision Train Marshalling Problem(DTMR)
Given a TMP instance (n, t, D) and $k \in N$, determine if there exists a k-solution.

Theorems

Theorem 1. A TMP instance $(n,\ k)$ admits a k-solution if and only if the directed graph $G(n, t, D, k)$ contains an N_{t+1}-rainbow path from node 0 to node $nk + 1$.
Theorem 2 (principle of exclusion-inclusion). Let U be a finite set and $P_1, \dots, P_t$ subsets of U. Then $|P_1 \cap \cdots \cap P_t| = \sum_{T \subseteq N_t} (-1)^{|T|} |\cap_{j \in T} \overline{P}_j|$, where $\overline{P}_j = U \setminus P_j$ and $\cap_{j \in \emptyset} \overline{P}_j = U$.
Lemma 1. Let (n, t, D) be a TMP instance with two consecutive cars i and $i + 1$ having the same destination. Then instance $(n - 1, t, D')$ obtained by removing car i has the same optimal value.
Theorem 3. Every TMP instance $(n,\ t,\ D)$ can be solved by a procedure requiring $O(\overline{n}Lt)$ space and $O(\overline{n}t^2 2^t L \log_2 L)$ time where $L = \min\{t, \lceil \overline{n}/4 + 1/2 \rceil\}$ and $\overline{n}$ is the number of cars in the contracted instance $C(n, t, D)$.