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presented:

Small feedback vertex sets in planar digraphs

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https://arxiv.org/pdf/1606.04419.pdf

Abstract

Let G be a directed planar graph on n vertices, with no directed cycle of length less than $g \geq 4$. We prove, that G contains a set X of vertices such that $G - X$ has no directed cycle, and $|X| \leq \frac{5n-5}{9}$ if $g = 4$, $|X| \leq \frac{2n-5}{4}$ if $g = 5$, and $|X| \leq \frac{2n-6}{g}$ if $g \geq 6$.

Definitions

- *Digirth* of a digraph G is the minimum length of a directed cycle in G (∞ if G is acyclic).
- A *feedback vertex set* in a digraph G is a set X of vertices such that $G - X$ is acyclic, and the minimum size of such set is denoted by $\tau(G)$.
- In a planar grapg, the degree of face F, denoted by $d(F)$, is the sum of the lengths (number of edges) of the boundary walls of F.
- $\mathcal{C}$ denotes a maximum collection of edge-disjoint directed cycles in G. If we fix a planar embedding of G, given a directed cycle C of $\mathcal{C}$, we denote by $\overline{C}$ the closed region bounded by C and by $\mathring{C}$ the interior of $\overline{C}$.
- For a cycle C in $\mathcal{C}$, we define the closed region $\mathcal{R}_C$ as $\overline{C}$ minus the interior of all cycles inside the interior of C.
- ϕ is the sum of $3d(F) - 6$, over all faces F of G.
- ϕ_C is the sum of $3d(F) - 6$, over all faces F of G lying in $\mathcal{R}_C$.

Theorems

Theorem 1. Prove the statement in abstract for all $n \geq 3$.

Lemma 2. Let H be a planar bipartite graph, with bipartition (U, V), such that all faces of H have degree at least 4, and all vertices of V have degree at least 2. Then H contains at most $2|U| - 4$ faces of degree at least 6.

Lemma 3. Let G be a connected planar graph, and let $S = \{F_1, \dots, F_k\}$ be a set of k faces of G, such that each F_i is bounded by a cycle, and these cycles are pairwise vertex-disjoint. Then

$$\sum_{F \notin S} (3d(F) - 6) \geq \sum_{i=1}^{k} (3d(F_i) + 6) - 12$$

where the first sum varies over faces F of G not contained in S.

Theorem 4. Let C_0 be a node of $\mathcal{F}$ (explained during presentation) with children $C_1, \dots, C_k$. Then $\phi_{C_0} \geq \frac{3}{2}(g-2)k + \frac{3}{2}g$. Moreover, if $g \geq 6$, then $\phi_{C_0} \geq \frac{3}{2}(g-2)k + \frac{3}{2}g + 3$.